where C is the consumption of a bundle of private goods and θ (< 1) is the elasticity of substitution. The budget constraint is:

$$C = Y - \frac{G}{N} \qquad (2)$$

with Y as the exogenous level of individual income.
It can easily be shown that the maximization of (1) subject to (2) yields the following optimal supply of the public good:

$$G = \frac{Y\,N}{N^{\frac{\theta}{\theta-1}} + 1} \qquad (3)$$

What we are interested in is the ratio of public expenditure to GDP, G/YN. If the first derivative of G/YN with respect to N,

$$\frac{\partial \frac{G}{Y\,N}}{\partial N} = \frac{\theta}{\theta-1}\,\frac{N^{\frac{1}{\theta-1}}}{(N^{\frac{\theta}{\theta-1}} + 1)^2} \qquad (4)$$

is negative, then the ratio of public expenditure to GDP declines with a growing number of tax payers or inhabitants, respectively.

Needless to say, the actual sign of equation (4) depends on the range of θ. The less substitutable C and G (the smaller θ), the greater the effect of population on government size. At the limit (where $\theta = -\infty$) one can easily see, though, that an increase of N runs in the opposite direction, but the right-hand side of equation (4) remains negative in any case, with the notable exception of $\theta = -\infty$ and $N = \infty$, where there is no influence of country size (population) on government size (public expenditure to GDP ratio). It is also fairly easy to show that there are no size effects by using a Cobb-Douglas utility function, which is approached by a unit elasticity of substitution here (θ = 0). For the interval θ = [0;1[we obtain a positive sign for the right-hand side of equation (4), which would be contrary to the conjecture developed above. If θ = 1, the utility function is linear, but equation (4) is not defined in that case.

According to Alesina and Wacziarg (1998) the resemblance to two well-known effects in economics provides a proper intuition. First, an increase in N of course reduces the per capita costs of provision of G and allows more income to be allocated to private consumption. This may be